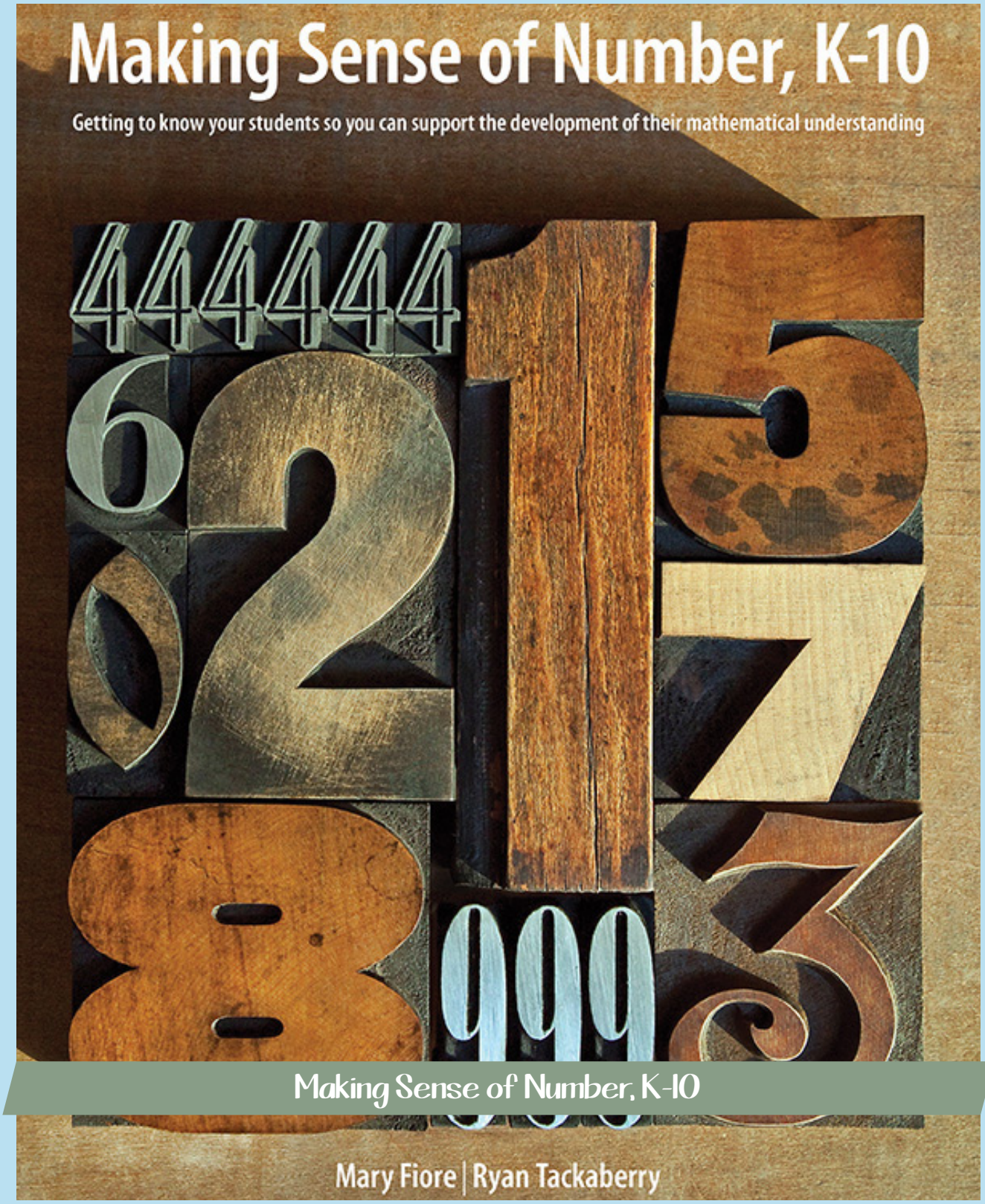
FRIDAY FREEBIE



The Components of Mathematical Proficiency

The following graphic organizers, or Frayer models, adapted from the National Research Council (2001), synthesize our understanding of the various components of mathematical proficiency.

Conceptual Understanding

Description: the comprehension and connection of concepts, operations, and relations

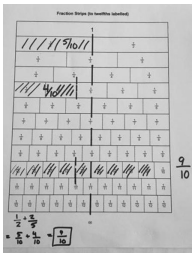
Essential Characteristics:

Students:

- understand the importance of a mathematical idea and in which contexts it is useful
- learn facts and methods and/or procedures
- organize their knowledge into a coherent whole
- represent mathematical ideas in different ways and know how different representations can be useful for different purposes

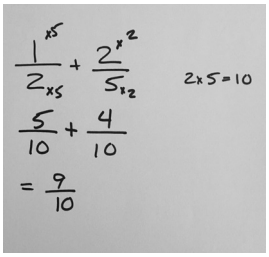
What it is:

- Find the sum of $\frac{1}{2} + \frac{2}{5}$.



What it isn't:

- Find the sum of $\frac{1}{2} + \frac{2}{5}$.



Note: The solution for "What it isn't" represents one example of student thinking. On its own, it does not necessarily represent a conceptual understanding. Rather, it demonstrates the use of an algorithm. We would have to gather other evidence of student thinking and understanding (e.g., something we noticed or something we heard while engaging in a conversation with the student).

Procedural Fluency

Description: the meaningful and flexible use of procedures to solve problems

Essential Characteristics:

Students:

- understand procedures and know when and how to use them appropriately
- perform procedures flexibly, accurately, and efficiently

What it is:

- effectively using mental math strategies (e.g., 6×8 is the same as 6×4 doubled)
- choosing an operation and strategy based on the situation and numbers
- estimating reasonably (e.g., I know that 22 + 19 is about 40 because 20 + 20 is 40)

What it isn't:

- memorizing rote facts
- performing calculations with large numbers by hand
- applying a standard algorithm without understanding

Description: the ability to formulate, represent, and solve mathematical problems

Essential Characteristics:

Students:

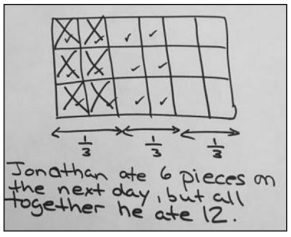
- generate a mathematical representation of the problem and ignore irrelevant features
- identify the mathematics they need to solve the problem
- understand various strategies and models

Strategic Competence

What it is:

- representing a problem accurately
- identifying key features to ensure the problem is understood

Jonathan ordered a pizza with 18 slices. The next day $\frac{2}{3}$ of the pizza remained. Jonathan then eats $\frac{1}{2}$ of what remains. How many pieces did he eat?



What it isn't:

- "number grabbing" (e.g., selecting numbers and performing operations)
- relying on a previous experience that used an algorithm and performing what is recalled and memorized

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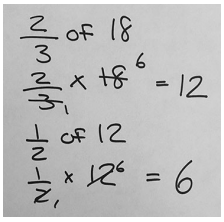

Components of Mathematical Proficiency

Adaptive Reasoning

Description: the capacity to think logically about the relationships among concepts and situations

Essential Characteristics:

Students:

- justify conclusions
- make sense of mathematical facts, procedures, concepts, and strategies and/or methods
- monitor their own progress and generate alternative plans if the current plan is ineffective

What it is:

- inductive reasoning (e.g., identify patterns by considering multiple instances)
- intuitive reasoning
- making inferences (e.g., student draws meaning from explicit and implicit information using prior knowledge and experiences)
- reasoning and proving mathematical thinking

What it isn't:

- justifying procedures only once (e.g., it isn't adaptive reasoning if students can only practise problems on multiplying fractions after the procedure has been introduced)

Productive Disposition

Description: the tendency to see sense in mathematics, to perceive it as both useful and worthwhile

Essential Characteristics:

Students:

- believe that steady effort in learning mathematics pays off
- see themselves as effective learners and doers of mathematics

What it is:

- viewing mathematics as understandable and not arbitrary
- displaying positive attitudes and beliefs about yourself as a mathematics learner

What it isn't:

- relying on memorizing rather than sense-making to solve problems
- viewing mathematical ability as fixed (e.g., not everyone can do math)
- viewing test questions and results of tests as the only measures of ability